Supermodels: Dynamically Coupled Imperfect Models

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In weather and climate prediction studies multi-model ensemble mean predictions are often employed to improve prediction skills (Tebaldi & Knutti, 2007). In the standard multi-model ensemble approach, the models are integrated in time independently and the predicted states are combined a posteriori. Recently an approach has been proposed in which the models exchange information during the simulation (van den Berge et al., 2011). This approach is called the supermodeling approach (SUMO). It assumes M imperfect models labeled by μ, each describing the dynamics of the model state vector $\boldsymbol{x}_\mu$ according to $\dot{x}^i_\mu = f^i_\mu(\boldsymbol{x}_\mu)$, in which i labels the vector components. The individual models μ are combined into one supermodel by prescribing nonnegative connections $C^i_{\mu\nu}$ between the i-th component of model μ and model ν,

$$\dot{x}^i_\mu = f^i_\mu(\boldsymbol{x}_\mu) + \sum_\nu C^i_{\mu\nu}(x^i_\nu - x^i_\mu)\,. \tag{1}$$

The connections are to be optimized using data, for instance by minimizing short term prediction error of the connected ensemble mean on the training data. Its potential has been demonstrated in the context of 3-D chaotic dynamical systems. With optimized connections, the models synchronize on a common solution that is closer to the true system than any of the individual model solutions.

In (Wiegerinck et al., 2013a), we have shown that with large connections, the SUMO follows approximately the weighted averaged trajectory

$$\dot{x}^i = \sum_\mu w^i_\mu f^i_\mu(\boldsymbol{x})\,. \tag{2}$$

where the weights $\{w^i_\mu\}$ can be derived from eigenvectors of the connection matrices. Also, with (2), we could understand local minimima in the connection space and results due to parameter perturbations reported in (van den Berge et al., 2011).

In (Wiegerinck et al., 2013b), we defined the supermodel directly according to (2), which we called weighted SUMO (opposed to connected SUMO in (1)). While connected SUMO needs nonlinear optimization, weighted SUMO can be optimized using linear optimization methods, making the method scalable for models of higher dimensions. We demonstrated the method in the context of a two-level, hemispheric, quasi-geostrophic spectral model on the sphere, triangularly truncated at wave number five, with 30 degrees of freedom (Houtekamer, 1991).

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